

MIT Press • Open Encyclopedia of Cognitive Science

Numerical Cognition

David Barner¹

¹Professor, Department of Psychology, UC San Diego

MIT Press

Published on: Nov 20, 2025

DOI: <https://doi.org/10.21428/e2759450.44f29f6b>

License: [Creative Commons Attribution-NonCommercial 4.0 International License \(CC-BY-NC 4.0\)](#).

Number is everywhere in human cognition, behavior, and culture. In a glance, a human observer can tell whether a stadium contains more fans for the home team or the visitors. When we want to be precise, we can count or use written numerals to express exact number. Number even gets expressed grammatically, when we use plural nouns or words like “many” or “a few.” These diverse abilities allow humans to describe and reason about abstract phenomena like time and space, to model the structure of invisible particles, or to imagine alternative universes. Some human numerical abilities are shared with other species, like mice and fish, whereas some are unique to humans. Some abilities are unique to only a handful of humans whose interest is drawn to the creation of formal systems for modeling our world.

History

The human capacity to reason numerically is as old as we are. There is evidence of numerical thought in archeological artifacts dating back at least 10,000 years and perhaps as far back as 20,000 years. Also, humans have wondered about what numbers are and how we come to know them for as long as we have been asking questions, and number has played an important role in the philosophy of mind for hundreds of years ([Shapiro, 2000](#)). Numbers pose a particularly interesting philosophical problem because they seem to have a life of their own outside of the human mind, but they are also not physical or directly observable in any way. Also, some mathematical truths appear to be immutable and necessary—such as $1 + 1 = 2$ —and yet we still need to learn them.

Until the 20th century, questions about numerical cognition were mostly discussed by philosophers, but things changed with the birth of psychology and the introduction of experimental methods to study cognition and behavior. Scientific reports on numerical cognition began with reports by Francis [Galton \(1880\)](#) and Wilhelm [Wundt \(1904\)](#) and in studies of birds and rodents ([Koehler, 1950](#); [Mechner, 1958](#)). The study of numerical cognition in humans took off in the 1970s, with the advent of the cognitive revolution, which shone new light on basic cognitive processes including numerical cognition. Early work by developmental psychologists such as [Piaget \(1953\)](#) and [Gelman and Gallistel \(1978\)](#) was especially influential [see [Cognitive Development](#)]; this work helped to spawn multiple new fields of inquiry that have investigated different age groups, cultures, animal species, and experimental methods.

Core concepts

Number sense and object tracking

All humans—regardless of education level or age—possess an ability to judge the relative number of things in two groups without counting. For example, if an image of 50 apples is quickly flashed followed by a second image of 30 apples, most people can judge that the first image contained more apples. A large body of literature suggests that this ability is supported by an evolutionarily ancient *approximate magnitude system*, which has been documented in pigeons, mice, fish, nonhuman primates, and humans across cultures ([Brannon, 2005](#); [Cantlon, 2012](#); [Dehaene et al., 1998](#)). When we use this system to compare

two groups of things, our success in identifying the larger group is determined by the ratio between them. Judging that 50 is more than 30 is relatively easy for most people, whereas 40 versus 30 is harder, and 35 versus 30 is nearly impossible. This is because our representation of each individual group of things is uncertain and approximate; when we see 50 things, sometimes we might think there are as few as 30 and other times 70 or more (see [Figure 1](#)). That is, numerical judgments using this system are ratio governed: comparing 50 versus 30 is no easier or harder than comparing 100 versus 60.

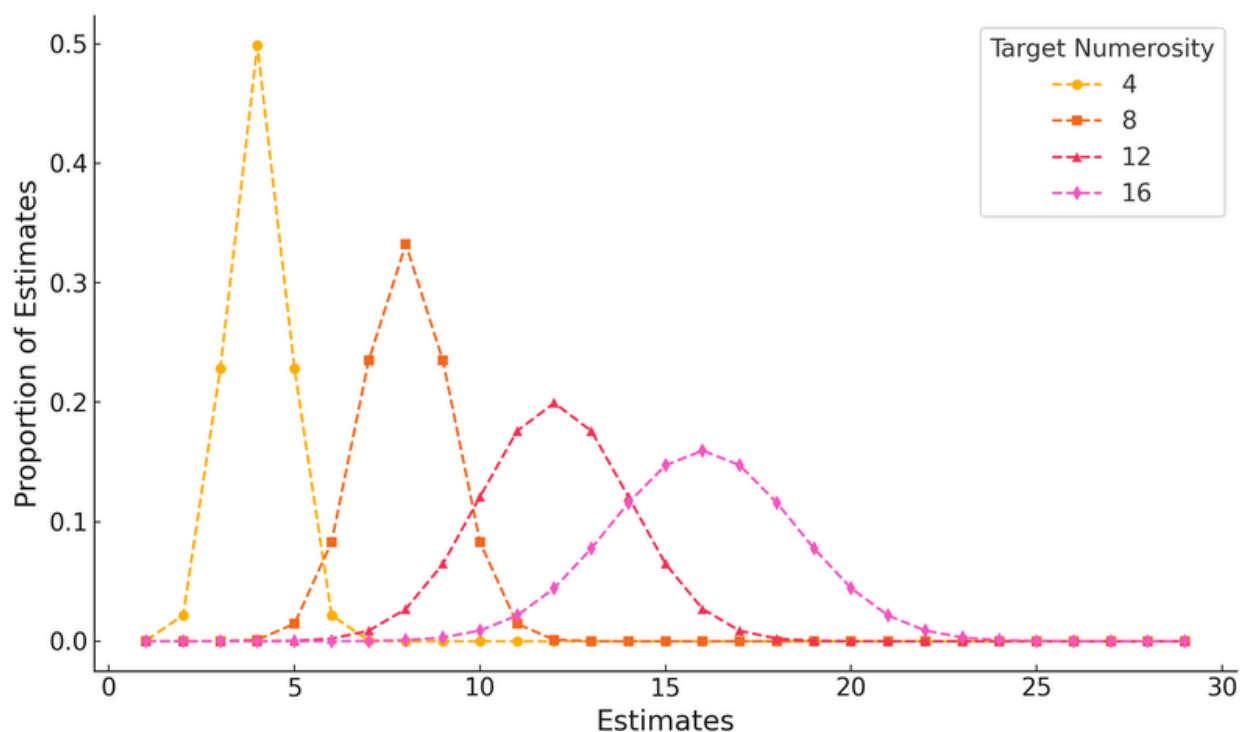


Figure 1

Approximate number estimates for groups of 4, 8, 12, and 16. Each line represents the distribution of verbal estimates that are typical when a human participant is presented with a target numerosity (e.g., 4, 8, 12, or 16 dots). Typically, the most common response is centered around the target, and responses are more accurate and less distributed for smaller numbers relative to larger ones.

There is now strong evidence that approximate magnitude representations have dedicated real estate in the brain and, in particular, the intraparietal sulcus ([Gallistel, 2021](#); [Nieder, 2021](#)). Similar findings have been reported in young infants ([Visibelli et al., 2024](#)) and in older children as well ([Piazza et al., 2006](#)). Finally, there is ample evidence that specialized areas are also used to represent number in other animal species ([Dehaene et al., 1998](#); [Nieder, 2021](#)). All of this suggests that, whatever this ability is, it is supported by an evolutionarily ancient biological system that is found across the human lifespan and across different species.

Interestingly, people get better at judging relative number as they age and gain experience with formal education ([Libertus et al., 2016](#)). Early in life, human infants can differentiate quantities that are in a 2:1 ratio but struggle to differentiate a 2:3 ratio ([Xu et al., 2005](#)). By preschool, a 2:3 ratio is easy for most

children, and by high school, they can differentiate groups in a 7:8 ratio. Meanwhile, members of innumerate groups (who do not have verbal counting systems or written numbers) like the Piraha and Mundurucu are able to differentiate quantities that exhibit relatively large ratios but are less likely than numerate adults to differentiate smaller ratios ([Gordon, 2004](#); [Pica et al., 2004](#)). Such differences may be due to differences in motivation, practice taking tests, or perhaps training with number words and counting.

The emergence of numerical symbols and artifacts in human culture

Humans use a variety of symbolic forms to represent number, including number words (e.g., one, two, three), verbal counting, body count systems, written numerals, and physical artifacts like counters and abacuses ([Ifrah, 2000](#)). There is some debate about when humans first began using external symbols to represent exact numbers [see [Cognitive Artifacts](#)]. On some accounts, the first numerical artifact is the Ishango Bone, a 20,000-year-old object discovered in the Democratic Republic of Congo that features a series of marks that correspond to prime numbers ([de Heinzelin, 1962](#)). However, others have speculated that the marks may have no mathematical significance and might even have been made by idle hands killing time or to improve grip of the bone. More certainty surrounds clay tokens dating back 10,000 years in current-day Iraq ([Schmandt-Besserat, 2010](#)). Small objects, sometimes formed into the shape of animals or common objects, were baked into clay envelopes, likely as a way to record economic transactions. These objects were sometimes used to make impressions on the clay envelopes before they were baked—like a kind of proto-writing system—and eventually just these impressions in clay were used to record numbers and featured symbols for large numbers in addition to the tokens representing individual things.

In addition to clay tokens, many cultures also created devices for counting and calculation, such as the abacus, which is still used to this day. A Japanese soroban abacus ([Figure 2](#)) features a series of vertical rods on which beads can be moved. Beads on each rod are divided into “earthly” beads near the bottom and “heavenly” beads at the top. On the rightmost vertical rod, each earthly bead represents a value of one, whereas each heavenly bead has a value of five. Moving one rod to the left, each bead’s value is multiplied by 10 so that earthly beads are worth 10, and heavenly beads are worth 50. The next rod to the left represents multiples of 100, and the next, multiples of 1000, and so on. To represent a specific number on a soroban, beads in the heavenly and earthly realms are moved toward one another to the horizontal rod that divides them, such as in [Figure 2](#), in which gray beads represent 28,918. Notably, numerous other abacus systems have been used over human history ([Ifrah, 2000](#)), many of which use a similar strategy of clustering counters into smaller groups that make them easier for human users to keep track of. These include the Chinese abacus (which features five earthly beads and two heavenly beads per column), the Russian schoty (which organizes beads horizontally rather than vertically), and the Roman hand abacus.

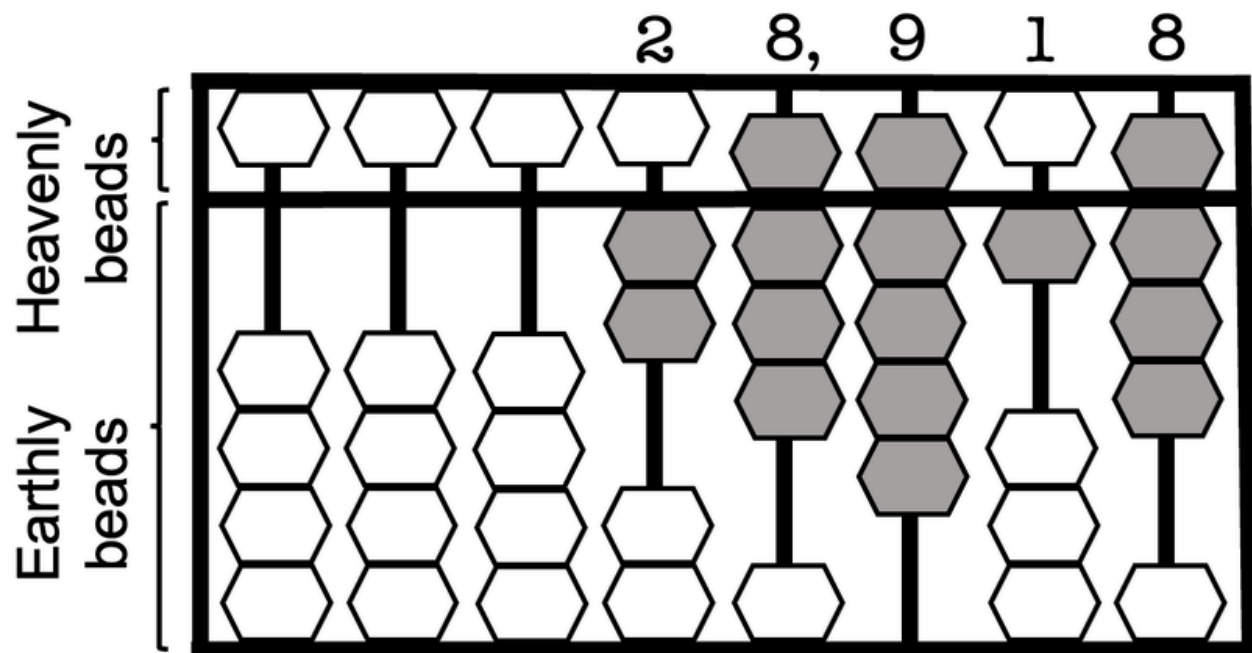


Figure 2

A soroban abacus representing the value 28,918. Gray represents beads that are in play and thus “count,” whereas white represents beads that are out of play.

In addition to these three-dimensional representations of number, humans have also created a large variety of different written numeral systems over human history (e.g., [Chrisomalis, 2020](#); [Ifrah, 2000](#)). Familiar to most people is the Hindu–Arabic positional or “place value” system, in which a numeral like 5 represents different quantities depending on its position in a numeral, much like the abacus. For example, in a number like 55, the first 5 represents 50, and the second represents 5. However, many other systems were invented over time. Some used tally marks to represent numbers (e.g., using IIII to represent four) and some systems even grouped marks into chunks of four to five to represent larger numbers (e.g., IIII IIII to represent eight). Other systems, like the familiar Roman numerals still in use today, added special symbols to represent multiples of these numbers, using a character like X for 10 or V for 5.

Numerical systems have also used a variety of base structures. Whereas the Hindu–Arabic system uses a base of 10 (e.g., including numbers like 20, 30, 40, etc.), other bases are also historically attested, including base 4, 5, and 20 and even base 60. In some cases, such bases likely emerged from concrete artifacts that were used to represent number in different cultures. For example, it is widely believed that base 10 counting systems emerged from the fact that humans have two hands totaling 10 fingers. Similarly, some have speculated that base 60 systems, which were first created by the Sumerians in 3100 BCE, may have evolved from a finger-counting system, in which the thumb of one hand was used to count each of the 12 bones comprising the four fingers of the hand, whereas the five fingers on the second hand were used to keep track of each count to 12—resulting in a maximum of 60 ([Ifrah, 2000](#)).

The emergence of number words and verbal counting

In parallel with the emergence of written numeral systems, humans have also created a variety of verbal systems to express number ([Corbett, 2000](#)). Early in human history, it is likely that most of these systems were highly limited and expressed only smaller numbers exactly while using approximate quantity words to express larger amounts. For example, although number words and counting are common ways to express number verbally, we also use forms like singular and plural nouns (e.g., a cup versus some cups), number agreement (e.g., these, those), and quantifiers (e.g., several, many) to express numerical information. Some current-day cultures have these kinds of grammatical number marking, plus specialized markers for quantities up to three. For example, some dialects of Slovenian mark singular and plural on nouns, like English does, but also include a dual marker when speaking about sets of two ([Almoammer et al., 2013](#)). Similar marking is also used in Arabic, and some languages may even have dedicated plural marking for sets of three ([Corbett, 2000](#)).

In fact, some languages only have small numbers. For example, in the Hup dialect of the Nadahup language spoken in the Brazilian Amazon, the word for one is “unity,” the word for two is “eye quantity,” and the word for three is “rubber tree seed quantity” ([Epps, 2006](#)). However, words for larger exact quantities do not exist. Other Nadahup dialects add to this a body counting system and accompany gestures with expressions like “both hands” to represent 10 and “both hands and both feet” to represent 20 (see [Figure 3](#)).

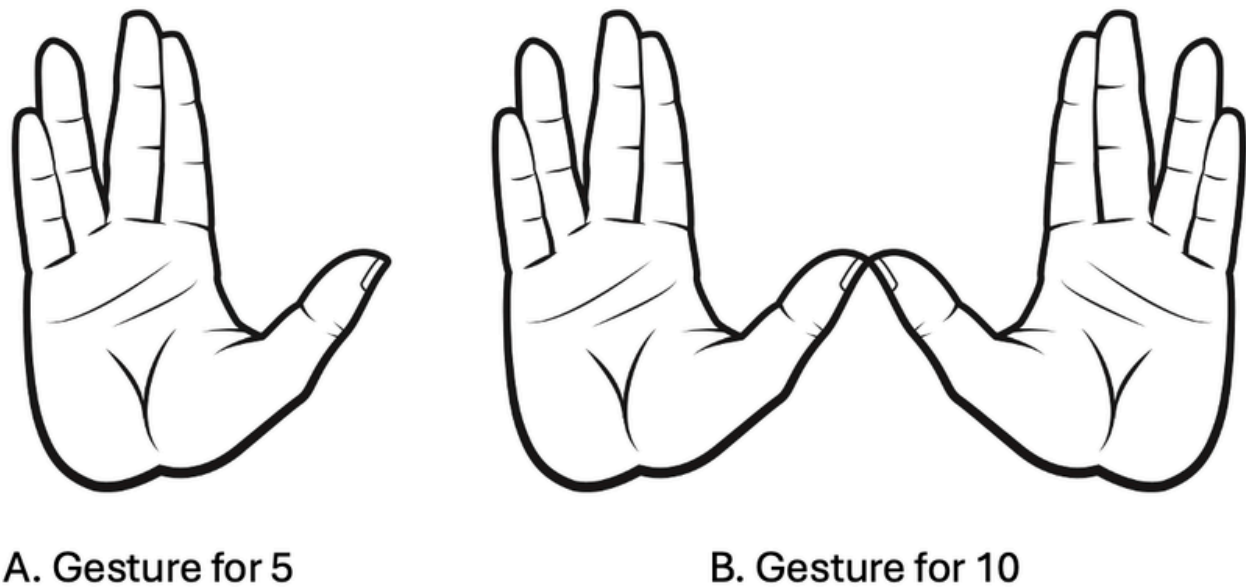


Figure 3
Nadahup (Dâw dialect) body count representations of five and 10.

Much more common in the modern era are verbal counting systems like the one used in English, which includes a series of simple rules that allow us to generate very large, although not infinitely large, numbers. English uses a base 10 verbal system, in which the words for one through nine are glued onto

bases like “twenty,” “thirty,” “forty,” etc. to create ever larger numbers. Additional bases occur at 100, 1,000, etc. Even the bases reflect a rule structure—“twen-ty” is 2×10 , and “thir-ty” is 3×10 . This can be seen much more transparently in languages like Cantonese, in which the word for three is “sàam,” the word for 10 is “sahp,” and the word for 30 is “sàam-sahp.” Although these base 10 structures are familiar to many in the Western world, like in the case of writing, verbal number systems have also used many different bases historically, most ranging from two to 20. These days, however, most humans use systems that use some version of base 10, sometimes mixing in remnants of older bases, as in the case of the French “quatre vingt” (four 20s) to represent 80.

A large literature describes how children learn to count and finds surprising limits to their early abilities across a range of cultures and languages (Carey & Barner, 2019) [see [Word Learning](#)]. By as early as age 2, many children—both in the United States and in other cultures—can recite a small set of number words in a rote routine, counting up to three, or five, or as high as 10. However, they rarely know what these numbers mean at this time or how to use them to represent exact number (Wynn, 1992). For example, when shown a group of three objects and asked how many there are, children at this age often guess randomly, despite being able to say the word three when counting. Also, when asked to give three objects, they give a random handful. In fact, children appear to learn the meanings of number words very slowly and initially one at a time over the course of many months. Typically they begin by learning the meaning of the word “one.” Then several months later, they learn the meaning of the word “two.” And again, several months after that, they learn “three” and sometimes “four” shortly thereafter. Surprisingly, however, none of these children can use counting to label and construct larger sets, despite being able to recite the counting list in speech.

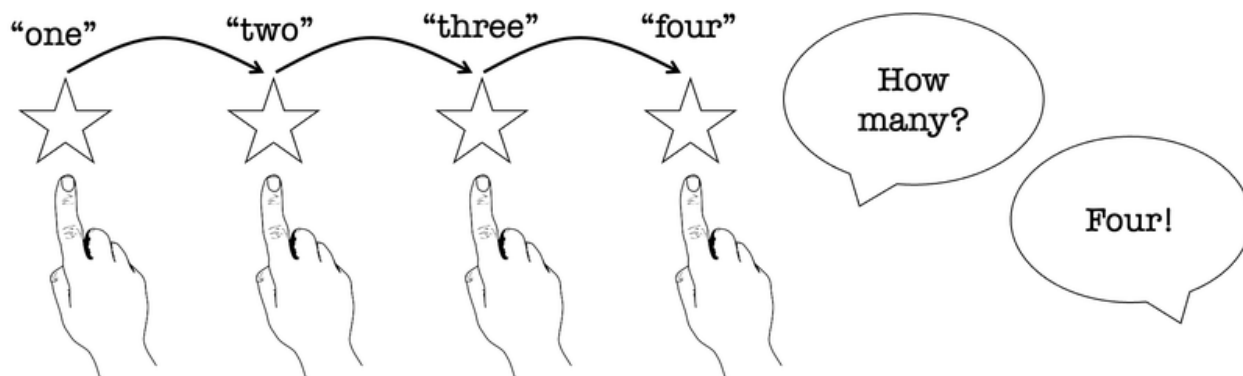


Figure 4

The use of one-to-one correspondence to accurately count large numbers.

By around the age of $3\frac{1}{2}$ or 4, many children begin to use counting in a more adult-like way, counting up from “one” to “two” to “three” while pointing at objects they are counting, keeping a tight one-to-one correspondence between number words and objects (see [Figure 4](#)) so that they always arrive at the same number when counting a set (Carey & Barner, 2019; Gelman & Gallistel, 1978; Wynn, 1992). These children can use this procedure to give sets larger than three to four and seem to have understood certain principles of counting. However, even then their knowledge is limited. For example, initially, when such a

child is told that there are five objects in a cup and then sees one more object added, they choose randomly when asked whether there are now six or seven objects ([Schneider et al., 2020](#)). Some of these limitations continue until age 5 or 6 and sometimes beyond. For example, many 5-year-old children do not know that if two sets are placed in one-to-one correspondence and therefore have the same number of items, that the same number word should be used to label both but that this is not true if one-to-one correspondence is disrupted ([Le et al., 2025](#)). Also, many young children claim that numbers are finite and end at some specific number, like 100, and only believe that numbers never end by 5½ or 6 years of age ([Cheung et al., 2017](#)). Finally, children are slow to associate their verbal numbers with perceptual, approximate representations of numbers and only begin to accurately estimate quickly presented groups of things after age 5 or 6 ([Sullivan & Barner, 2014](#)).

Crucially, important individual differences exist between children during early stages of numerical development, which are predictive of later classroom achievement (e.g., [Geary et al., 2018](#)). A wide range of factors appear to influence early learning. For example, delays in early learning outcomes are associated with developmental neurocognitive differences such as loss of hearing ([Santos & Cordes, 2022](#)). They are also related to parental education, parental anxiety about mathematics, access to quality preschool instruction and education, and socioeconomic status ([Sarnecka et al., 2023](#)). Some studies find that parental talk about counting and labeling sets is associated with children's ability to accurately count large sets of objects ([Gunderson & Levine, 2011](#)) and may even impact later math achievement ([Purpura et al., 2019](#)). Also, other math practices, like teaching children to count with the fingers, may also impact learning ([Goldin-Meadow et al., 2014](#)). Some children experience particular difficulties thinking and talking about number and are diagnosed with dyscalculia ([Butterworth, 2010](#)). Consequently, novel training paradigms and educational interventions continue to be developed and revised to enhance mathematical outcomes for students (e.g., [Hawes et al., 2021](#)).

Questions, controversies, and new developments

Previous work has documented the diverse ways in which humans engage with number, from written symbols, to physical calculators, to counting, number words, and grammatical expressions of quantity, although much descriptive work remains to be done. Still, despite this wealth of knowledge, many important puzzles remain. One of these is how and why humans created symbolic number systems in the first place, some 10,000 years ago. Although researchers have a good idea of what these systems were used for, it remains a mystery how the earliest innovations came about and whether they were pioneered by individual strokes of genius or emerged organically from cultural practices, without particular moments of insight. Recent work suggests that some progress on such questions might be made by creating experimental simulations of cultural evolution, although such work is in its infancy (e.g., [Holt et al., 2024](#)) [see [Cultural Evolution](#)].

A related question is whether such symbolic abilities are restricted to humans. Although we know that no other animals have created symbols for large exact number, some animals, like Ai the chimpanzee, seem

able to interact with human symbols in semihuman ways ([Matsuzawa, 1985](#)). It remains debated whether the propensity for language explains why only humans are sensitive to exact differences in large numbers ([Gordon, 2004](#); [Pica et al., 2004](#); [Schneider et al., 2022](#)). Alternatively, it may be that our organization into large cultural groups that engage in trade, sharing, lending, and debt is more important ([Graeber, 2014](#)).

Finally, much more remains to be known about how human cognition changes when we learn symbolic number systems and whether this fundamentally changes how we understand number—bringing with it entirely new concepts—or if it instead reorganizes concepts that humans have available to them at birth ([Carey & Barner, 2019](#)). Some researchers argue that core numerical concepts, including mental representations of the positive integers, are innate and represented by the approximate magnitude system ([Clarke & Beck, 2021](#); [Gallistel, 2021](#)). On some accounts, these representations are organized spatially or in a format similar to a number line (e.g., [Siegler & Opfer, 2003](#)). Some of this evidence comes from studies of the spatial-numerical association of response codes effect, which finds that people are faster at making numerical judgements when smaller numbers appear on the left and larger numbers appear on the right ([Dehaene et al., 1993](#)). Other researchers are more circumspect about whether approximate magnitudes represent number at all and argue that most results in the literature can be explained by nonnumerical features. For example, they suggest that deciding whether 50 or 30 is more might be calculated on the basis of the total area that each group takes up or some other feature that is correlated with number ([Leibovich et al., 2017](#)). These researchers often argue that there is no dedicated number system but instead a “generalized magnitude system” ([Lourenco, 2015](#)). Still others argue that any such ability to perceive quantity approximately is not truly numerical at all and that the word “number” should be reserved for culturally mediated practices like counting and arithmetic ([Núñez, 2017](#)).

Adding to these controversies is the idea that there may actually be more than one nonverbal number system. Early studies of numerical perception noticed that participants perform much more quickly and accurately when comparing small sets than when comparing larger ones. For example, human adults are more or less perfect in judging that three is more than two and do so almost instantly ([Mandler & Shebo, 1982](#)). Picking up on this idea, subsequent work argued that, beginning in infancy, humans as well as nonhuman animals represent small sets using an object-tracking system ([Feigenson et al., 2004](#)). Crucially, this system does not sum objects into magnitudes but is thought to represent each object individually, with a limit of up to three to four items. As evidence for this, when preverbal infants watch as one, two, or three objects are hidden in a box, they search until they find them all but are confused when four objects are hidden, often only searching until they find one object and then giving up. Similar results have also been reported using other methods. For example, when infants see three crackers hidden in one bucket and one hidden in another, they prefer to crawl to the bucket with three crackers, but they fail when tested with larger numbers, like three versus four ([Feigenson et al., 2004](#)).

These studies, when combined with work on the approximate magnitude system, have sparked interesting debates about which systems are used for which tasks and how representations of objects differ from representations of magnitudes and number ([Cantrell & Smith, 2013](#)). Such debates are

important because they are critical to understanding where human concepts of number come from and how we eventually learn to think and talk about integers, rational numbers, real numbers, and other abstract numerical properties (Carey & Barner, 2019).

Broader connections

Number permeates cognition and human culture and is found not just in math classrooms but in basic perceptual processes, in economic exchanges, and in writing and linguistic systems, dating back thousands of years. Number is studied from every approach in the cognitive sciences, including linguistics, philosophy, psychology, neuroscience, anthropology, computer science, education, and mathematics, and therefore is among the most interdisciplinary fields of current inquiry. Among the most interesting opportunities for future work is to further deepen connections between these various approaches so that psychologists and neuroscientists may interact more with philosophers of mathematics, anthropologists who study numerical artifacts, and educators who seek to understand best practices for instilling an understanding of number in young learners.

Acknowledgments

The author thanks Urvi Maheshwari and Khuyen Le for their extensive comments on an earlier version of this manuscript. The author is supported by National Science Foundation Grant 2201960.

Further reading

- Chrisomalis, S. (2020). *Reckonings: Numerals, cognition, and history*. MIT Press.
- Kadosh, R. C., & Dowker, A. (Eds.). (2015). *The Oxford handbook of numerical cognition*. Oxford University Press.
- Samuels, R., & Snyder, E. (2024). *Number concepts: An interdisciplinary inquiry*. Cambridge University Press.

References

- Almoammer, A., Sullivan, J., Donlan, C., Marušič, F., Žaucer, R., O'Donnell, T., & Barner, D. (2013). Grammatical morphology as a source of early number word meanings. *Proceedings of the National Academy of Sciences*, 110(46), 18448-18453. <https://doi.org/10.1073/pnas.1313652110>

↩

- Brannon, E. M. (2005). What animals know about numbers. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 85-107). Psychology Press.

↩

- Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences*, 14(12), 534–541. <https://doi.org/10.1016/j.tics.2010.09.007>

↩

- Cantlon, J. F. (2012). Math, monkeys, and the developing brain. *Proceedings of the National Academy of Sciences*, 109(supplement_1), 10725-10732. <https://doi.org/10.1073/pnas.1201893109>

↩

- Cantrell, L., & Smith, L. B. (2013). Open questions and a proposal: A critical review of the evidence on infant numerical abilities. *Cognition*, 128(3), 331-352. <https://doi.org/10.1016/j.cognition.2013.04.008>

↩

- Carey, S., & Barner, D. (2019). Ontogenetic origins of human integer representations. *Trends in Cognitive Sciences*, 23(10), 823-835. <https://doi.org/10.1016/j.tics.2019.07.004>

↩

- Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive Psychology*, 92, 22-36. <https://doi.org/10.1016/j.cogpsych.2016.11.002>

↩

- Chrisomalis, S. (2020). *Reckonings: Numerals, cognition, and history*. MIT Press.

↩

- Clarke, S., & Beck, J. (2021). The number sense represents (rational) numbers. *Behavioral and Brain Sciences*, 44, e178. <https://doi.org/10.1017/S0140525X21000571>

↩

- Corbett, G. G. (2000). *Number*. Cambridge University Press.

↩

- de Heinzelin, J. (1962). Ishango. *Scientific American*, 206(6), 105-118. <https://doi.org/10.1038/scientificamerican0662-105>

↩

- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, 122(3), 371-396. <https://doi.org/10.1037/0096-3445.122.3.371>

↩

- Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences*, 21(8), 355-361. [https://doi.org/10.1016/s0166-2236\(98\)01263-6](https://doi.org/10.1016/s0166-2236(98)01263-6)

↩

- Epps, P. (2006). Growing a numeral system: The historical development of numerals in an Amazonian language family. *Diachronica*, 23(2), 259-288. <https://doi.org/10.1075/dia.23.2.03epp>

↩

- Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8(7), 307-314. <https://doi.org/10.1016/j.tics.2004.05.002>

↩

- Gallistel, C. (2021). The approximate number system represents magnitude and precision. *Behavioral and Brain Sciences*, 44, e187. <https://doi.org/10.1017/S0140525X21000959>

↩

- Galton, F. (1880). Visualised numerals. *Nature*, 21(533), 252-256. <https://doi.org/10.1038/021252a0>

↩

- Geary, D. C., vanMarle, K., Chu, F. W., Rouder, J., Hoard, M. K., & Nugent, L. (2018). Early conceptual understanding of cardinality predicts superior school-entry number-system knowledge. *Psychological Science*, 29(2), 191-205. <https://doi.org/10.1177/0956797617729817>

↩

- Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Harvard University Press.

↩

- Goldin-Meadow, S., Levine, S. C., & Jacobs, S. (2014). Gesture's role in learning arithmetic. In L. D. Edwards, F. Ferrara, & D. Moore-Russo (Eds.), *Emerging perspectives on gesture and embodiment in mathematics* (pp. 51-72). Information Age Publishing.

↩

- Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science*, 306(5695), 496-499. <https://doi.org/10.1126/science.1094492>

↩

- Graeber, D. (2014). *Debt: The first 5,000 years, updated and expanded*. Melville House.

↩

- Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Developmental Science*, 14(5), 1021-1032. <https://doi.org/10.1111/j.1467-7687.2011.01050.x>

↩

- Hawes, Z., Merkley, R., Stager, C. L., & Ansari, D. (2021). Integrating numerical cognition research and mathematics education to strengthen the teaching and learning of early number. *British Journal of Educational Psychology*, 91(4), 1073-1109. <https://doi.org/10.1111/bjep.12421>

↩

- Holt, S., Fan, J. E., & Barner, D. (2024). Creating ad hoc graphical representations of number. *Cognition*, 242, 105665. <https://doi.org/10.1016/j.cognition.2023.105665>

↩

- Ifrah, G. (2000). *The universal history of computing: From the abacus to quantum computing*. Wiley.

↩

- Koehler, O. (1950). The ability of birds to “count.” *Bulletin of Animal Behaviour*, 9, 41-45.

↩

- Le, K. N., Schneider, R. M., & Barner, D. (2025). The development of cardinal extension: From counting to exact equality. *Developmental Psychology*, 61(6), 1180-1195. <https://doi.org/10.1037/dev0001922>

↩

- Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2017). From “sense of number” to “sense of magnitude”: The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*, 40, e164. <https://doi.org/10.1017/S0140525X16000960>

↩

- Libertus, M. E., Odic, D., Feigenson, L., & Halberda, J. (2016). The precision of mapping between number words and the approximate number system predicts children's formal math abilities. *Journal of Experimental Child Psychology*, 150, 207-226. <https://doi.org/10.1016/j.jecp.2016.06.003>

↩

- Lourenco, S. F. (2015). On the relation between numerical and non-numerical magnitudes: Evidence for a general magnitude system. In D. C. Geary, D. B. Berch, & K. M. Koepke (Eds.), *Mathematical cognition and learning* (pp. 145-174). Elsevier.

↩

- Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, 111(1), 1-22. <https://doi.org/10.1037/0096-3445.111.1.1>

↩

- Matsuzawa, T. (1985). Use of numbers by a chimpanzee. *Nature*, 315(6014), 57-59.

<https://doi.org/10.1038/315057a0>

↩

- Mechner, F. (1958). Probability relations within response sequences under ratio reinforcement. *Journal of the Experimental Analysis of Behavior*, 1(2), 109. <https://doi.org/10.1901/jeab.1958.1-109>

↩

- Nieder, A. (2021). The evolutionary history of brains for numbers. *Trends in Cognitive Sciences*, 25(7), 608-621. <https://doi.org/10.1016/j.tics.2021.03.012>.

↩

- Núñez, R. E. (2017). Is there really an evolved capacity for number? *Trends in Cognitive Sciences*, 21(6), 409-424. <https://doi.org/10.1016/j.tics.2017.03.005>

↩

- Piaget, J. (1953). How children form mathematical concepts. *Scientific American*, 189(5), 74-79. <https://doi.org/10.1038/scientificamerican1153-74>

↩

- Piazza, M., Mechelli, A., Price, C. J., & Butterworth, B. (2006). Exact and approximate judgements of visual and auditory numerosity: An fMRI study. *Brain Research*, 1106, 177-188. <https://doi.org/10.1016/j.brainres.2006.05.104>

↩

- Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, 306 (5695), 499-503. <https://doi.org/10.1126/science.1102085>

↩

- Purpura, D. J., Napoli, A. R., & King, Y. (2019). Development of mathematical language in preschool and its role in learning numeracy skills. In D. C. Geary, D. B. Berch, & K. M. Koepke (Eds.), *Cognitive foundations for improving mathematical learning* (pp. 175-193). Academic Press.

↩

- Santos, S., & Cordes, S. (2022). Math abilities in deaf and hard of hearing children: The role of language in developing number concepts. *Psychological Review*, 129(1), 199-211. <https://doi.org/10.1037/rev0000303>

↩

- Sarnecka, B. W., Negen, J., Scalise, N. R., Goldman, M. C., & Rouder, J. N. (2023). The real preschoolers of Orange County: Early number learning in a diverse group of children. *Journal of Numerical Cognition*, 9(1), 65-88. <https://doi.org/10.5964/jnc.6577>

↩

- Schmandt-Besserat, D. (2010). *How writing came about*. University of Texas Press.

↩

- Schneider, R. M., Brockbank, E., Feiman, R., & Barner, D. (2022). Counting and the ontogenetic origins of exact equality. *Cognition*, 218, 104952. <https://doi.org/10.1016/j.cognition.2021.104952>

↩

- Schneider, R. M., Sullivan, J., Marušič, F., Biswas, P., Mišmaš, P., Plesničar, V., & Barner, D. (2020). Do children use language structure to discover the recursive rules of counting? *Cognitive Psychology*, 117, 101263. <https://doi.org/10.1016/j.cogpsych.2019.101263>

↩

- Shapiro, S. (2000). *Thinking about mathematics: The philosophy of mathematics*. Oxford University Press.

↩

- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, 14(3), 237-250. <https://doi.org/10.1111/1467-9280.02438>

↩

- Sullivan, J., & Barner, D. (2014). Inference and association in children's early numerical estimation. *Child Development*, 85(4), 1740-1755. <https://doi.org/10.1111/cdev.12211>

↩

- Visibelli, E., Porru, A., Lucangeli, D., Butterworth, B., & Benavides-Varela, S. (2024). Neural indicators of numerical abilities in the infant human brain: A systematic review. *Developmental Review*, 74, 101150. <https://doi.org/10.1016/j.dr.2024.101150>

↩

- Wundt, W. M. (1904). *Principles of physiological psychology*. Sonnenschein.

↩

- Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, 24(2), 220-251. [https://doi.org/10.1016/0010-0285\(92\)90008-P](https://doi.org/10.1016/0010-0285(92)90008-P)

↩

- Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science*, 8(1), 88-101. <https://doi.org/10.1111/j.1467-7687.2005.00395.x>

↩